An Efficient Finite-Element Approach for the Modeling of Planar Double-D Excitation Coils and Flaws in SQUID NDE Systems

Farrokh Sarreshtedari, Member, IEEE, Arash Pourhashemi, Narjes Asad, Juergen Schubert, Marko Banzet, and Mehdi Fardmanesh, Senior Member, IEEE

Abstract—Incorporating an efficient approach for the finite-element simulation of eddy current superconductive quantum interface device (SQUID) nondestructive evaluation (NDE) systems, an appropriate finite-element method (FEM) has been presented for simulating and analyzing such systems. We have introduced a new model for the planar double-D coils, which are used as the excitation source in eddy current SQUID NDE systems, and also another model for the description of the flaw effect on the induced current. We have also examined our simulation results with their associated measurements. Our system is based on a high-$T_C$ YBCO gradiometer RF-SQUID sensor with a flux noise level below $100 \mu \Phi_0 / \sqrt{Hz}$ at 100 Hz in an unshielded environment while being shielded against external RF electromagnetic interference. The very good agreement between experimental and numerical approaches confirmed our model for the 3-D FEM simulation of the system, which is being done in reasonable time and using reasonable computer resources.

Index Terms—Double-D modeling, finite element, flaw modeling, RF superconductive quantum interface device (SQUID) gradiometer, SQUID nondestructive evaluation (NDE).

I. INTRODUCTION

FINITE-ELEMENT modeling is an efficient approach for simulating and describing eddy current nondestructive evaluation (NDE) systems [1], [2]. In [3], we used our new models in the finite-element method (FEM) simulation of the system for optimization of different eddy current superconductive quantum interface device (SQUID) NDE system parameters with little emphasis on the selection of such models. In that work, it is stated that incorporating the FEM algorithm for the exact modeling of small flaws and cracks needs tremendous number of mesh elements and a great deal of computer resources. This is an essential limitation of the FEM approaches for the 3-D simulation of the whole SQUID NDE systems [1], [4], [5]. According to this limitation of FEM, we have divided the problem of the simulation into two phases, in which as

the first step the planar double-D coil is modeled as a solid object with a special current distribution, and consequently, the different components of the induced current in the sample are found. The penetration of the induced current in the sample obeys the induction law [4]. In the second phase of the simulation, we model the perturbation in current distribution caused by the defect as a new source of current and then find its associated magnetic field at the sensor position [6], [7]. In this paper, we describe in detail our models for planar double-D excitation coils and small flaws, and then by using experimental measurements, we examine our FEM numerical simulations by comparison with experimental results.

For the experimental setup, we have implemented a SQUID NDE system based on a first-order RF-SQUID gradiometer as the magnetic sensor, a planar double-D coil [8] that is made on a two-layer printed circuit board as the excitation source, a micropositioner for precise $x$, $y$, and $\theta$ adjustments of the excitation coil, and liquid nitrogen cryogenic systems, and also required lock-in detection equipment for phase-sensitive measurements. The detail of the measurement setup is explained in [3] and [9].

Fig. 1 shows the schematic of the gradiometer SQUID in the middle of a planar double-D coil, which is used in our
simulations and is also physically implemented for the experimental measurements. Considering the relative position of the flaw with respect to the coil, we first determine the direction and the amplitude of the induced current at the position of the flaw by incorporating a divided developed model for the double-D coil. In the second phase, we use another developed model for the description of the flaw affect as a perturbing source in the induced current.

II. MODELING OF THE DOUBLE-D COIL

The configuration of the excitation coil has a considerable effect on the response of the NDE system, and so different configurations have been proposed [8], [10], [11]. Among them, the double-D coil, which has a simple configuration, makes a zero magnetic field and a zero magnetic field gradient at its center that is absolutely desirable for using gradiometer SQUIDs. Fig. 2 shows the layout of the planar double-D coil designed on a printed circuit board. This pattern consists of two layered circular thin tracks, which produce a symmetric field such that it vanishes in the center of the coil that cannot be seen by the SQUID.

To model this configuration as our excitation coil in the FEM simulator, we approximate it with a combination of some simple regions that have uniform current densities in each. Instead of dealing with a discrete current path due to different tracks, we have considered a continuous current density covering the 2-D area of the coil. The distribution of the current is assigned uniformly circular in the curved track section shown in Fig. 1, but for the straight track section, we first approximate the straight section with a triangular shape.

Considering Kirchhoff’s current law, to relate the currents in the triangular section to the known current of the curved section, we used the fact that as the current with density $j_1$ reaches the boundary with angle $\theta_1$, it will be changed to $j_2$ with angle $\theta_2$, satisfying

$$\frac{j_1}{j_2} = \frac{\sin \theta_2}{\sin \theta_1}. \quad (1)$$

Using (1), as $\varphi$ changes between $0^\circ$ and $90^\circ$, the current density $j_2$ can be related to $j_1$ by

$$j_1 \left[ \sin \left( (90 - \varphi) - \alpha \right) \right] = j_2(y) \left[ \sin(\alpha) \right] \quad (2)$$

where $j_1$ is the constant current density of the curved section, and $j_2(y)$ is the $y$-dependent current density in the triangular section. As shown in Fig. 3, the angle $\varphi$ is related to $x$, $y$, and $\alpha$ by

$$\varphi = \tan^{-1}(y/x) = \tan^{-1}(y/(-y/tg\alpha + a)) \quad (3)$$

where $x$ and $y$ represent the coordinates of the points on the boundary line between two sections.

Fig. 4 shows the current density $j_2(y)$ as $y$ goes from $y = 0$ to $y = y_0$, where $y_0$ is the vortex of the triangle section in Fig. 3. The minus part of the current density $j_2(y)$ in Fig. 4 is related to those curved tracks that are included in the triangular section.

For approximating the triangular section, which contains the variable current density by some sections with fixed densities, we can divide the triangular section to $n$ arbitrary regions.

Fig. 5 shows the simulation results of dividing this section into two and three regions and comparing them with the result of applying the accurate relation for the current distribution of $j_2(y)$, as presented in Fig. 4. It can be inferred from these simulations that incorporating a three-region approximation would lead to a design with satisfactory accuracy.

Fig. 6 shows the approximated coil configuration in which the current densities $J_2$ and $J_3$ are calculated as in (4) and (5), shown at the bottom of the next page.
III. MODELING OF THE FLAW

As the eddy current in the sample encounters the flaw, its path deviates just like when an incompressible fluid flows around a circular disc. The similarity between these two phenomena makes it possible to use well-known hydrodynamic equations for the description of eddy current deformation [12].

Here, we have used the FEM simulation for investigating the induced current perturbations due to the flaws. Considering a slab aluminum sample with a small region of zero conductivity as the flaw in it and incorporating appropriate boundary conditions, the distribution of the induced current around the flaw can be found as shown in Fig. 8. By subtracting the yielded distribution in Fig. 8 from the uniform distribution of the induced current, the influence of the flaw as a perturbing source can be extracted. Fig. 9 shows this subtracted distribution current. The figure confirms that the effect of the flaw can be modeled as an electric current dipole just like a fluid dipole, which is considered as the effect of the fluid perturbation caused by a disc [7], [12], [13].

\[
J_2 = \frac{J_1}{y_0 - y_{\text{center}}} \times \int_{y_{\text{center}}}^{y_0} \sin \left( \pi - \arctan \left( y / \left( y / \tan(c) + a \right) \right) \right) / \sin(c) \, dy
\]  \tag{4}

\[
J_2 = \frac{J_1}{y_{\text{center}}} \times \int_{0}^{y_{\text{center}}} \sin \left( \pi - \arctan \left( y / \left( y / \tan(\alpha) + a \right) \right) \right) / \sin(\alpha) \, dy
\]  \tag{5}
This localized current source has a uniform current density interior of the flaw and a fading pattern around it. We have modeled this pattern as a new three-region element, with its schematic shown in Fig. 10.

In this modeling, considering the Maxwell equation of \( \nabla \cdot \mathbf{B} = 0 \), the current density of the interior circle is equal to the unperturbed current, and the current densities of the two concentric rings are approximated as the average of the perturbed current around the flaw. These current densities are found as

\[
J_2 = \frac{1}{(R_2 - R_1)} \int_{R_1}^{R_2} J_{\text{Field}} \, dr \\
J_3 = \frac{1}{(R_3 - R_1)} \int_{R_1}^{R_3} J_{\text{Field}} \, dr
\]

where \( R_1, R_2, \) and \( R_3 \) are shown in Fig. 10, and \( J_{\text{Field}} \) is the perturbed current around the flaw, which is shown in Fig. 9.

IV. RESULTS AND DISCUSSION

Incorporating the developed FEM approach for the simulation of the gradiometer SQUID readout of flaw effect in the eddy current NDE, we have simulated the system for different relative positions of the flaw and sensor. Fig. 11 shows the result as if we fix the SQUID position and revolve the position of the flaw along concentric circles with different radius around the center of the excitation coil.

Fig. 11 shows the periodic behavior of the response, the relative amplitudes of the detectable signal, and the positions of the flaw, which give maximum and minimum response. In this figure, \( R_0 \) is the coil radius, and the angle is measured with respect to the reference shown in Fig. 12. By further investigating such simulations, it can also be inferred that the peak value of the signal can be obtained when the flaw is placed at the polar coordinate of \((0.48R_0, 55^\circ)\) and also their corresponding periods. The right hand part of Fig. 12 shows the response of the system as the flaw moves along the radial line of AB, which contains the mentioned point of maximum response.

The outcome of this simulation for the determination of the relative flaw and sensor positions, which makes the maximum signal amplitude, is closely compatible with the experimental SQUID NDE scanning result. Using our SQUID NDE system, we have also examined the FEM-developed models with different experimental scanning results. Figs. 13 and 14 show the experimental scanning of a single hole in an aluminum plate and its counterpart simulation using the same frequency of 40 Hz. The system parameters for experimental measurements are set according to the optimization results in [3]. In that reference, the comparison between the FEM simulations and the experimental measurements is also presented. We have also used this approach for an enhancement in application of signal processing algorithms for defect detections [14].
Fig. 12. (Left) Flaw positions at different concentric circles with different radius. (Right) Simulation result of the system as flaw moves along AB.

Fig. 13. Experimental results of a 2-D SQUID-based NDE scanning of a sample with hidden flaw.

Fig. 14. Simulation results of a 2-D scanning of a sample with hidden flaw.

V. CONCLUSION

An efficient approach for the FEM simulation of the SQUID NDE system has been developed. Using novel models for the simulation of flaws and double-D excitation coils, we have simulated our system and compared the results with their corresponding experimental results. Incorporating the simulations and comparison with the measurements, it is shown that the three-region approximation of the double-D coils fits the experimental results very well. Based on the very good agreement between our FEM simulations and the experimental results, it has been concluded that the proposed simulation approach has the simultaneous features of high accuracy and consumption of the reasonable computation resources and also in reasonable time.

REFERENCES

Farrokh Sarrehtedari (M’10) was born in Iran in 1982. He received the B.S. and M.S. degrees in electrical engineering from Sharif University of Technology, Tehran, Iran, in June 2009. He is currently working toward the Ph.D. degree in electrical engineering.

He is currently a member of the Superconducting Electronics Research Laboratory (SERL), Department of Electrical Engineering, Sharif University of Technology. His research interests include superconductivity, SQUID devices, SQUID-based systems, and solid-state and photonic devices.

Arash Pourhashemi was born in Iran. He received the B.Sc. degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in June 2009. He is currently working toward the Ph.D. degree at the University of California, Santa Barbara.

He joined Sharif University of Technology, Tehran, Iran, in September 2004, where he worked on superconductive quantum interface device (SQUID)-based nondestructive evaluation (NDE) systems and superconducting edge transition bolometers during the last two years of his undergraduate studies. His current research with the Nanoelectronics Research Laboratory (NRL) is in the area of carbon nanoelectronics.

Narjes Asad was born in Iran in 1986. She received the B.S. degree in physics from the University of Tehran, Tehran, Iran, in 2009.

She is currently with the Superconductivity Research Laboratory, Department of Physics, University of Tehran, where she has worked on photoluminescence spectroscopy. Her recent projects were in the area of particle physics and modeling and simulation of passage of particles through matter. Her research interests include solid-state physics, superconductivity, and computational physics.

Juergen Schubert was born in Cologne, Germany, in 1958. He received the Diploma degree in physics and the Ph.D. degree from the University of Cologne, Cologne, in 1985 and 1989, respectively.

Since 1984, he has been with the Jülich Research Center (Forschungszentrum Jülich), Jülich, Germany. From 1985 to 1989, he developed a high-pressure sputter technique for the growth of high-temperature superconductor thin films. Since then, he has been the Leader of the Laserlab of the ISG 1-IT of the Jülich Research Center, and he is responsible for the growth of epitaxial oxide thin films (superconducting, ferroelectric, optical transparent, conducting, etc.) using the pulsed laser deposition method. In 2002, he spent one year as a Guest Scientist at Pennsylvania State University, State College, where he worked in the oxide molecular beam epitaxy group of Darrell Schlom.

Mehdi Fardmanesh (SM’02) was born in Tehran, Iran, in 1961. He received the B.S. degree in electrical engineering from Tehran Polytechnic University, Tehran, in 1987, and the M.S. and Ph.D. degrees in electrical engineering from Drexel University, Philadelphia, PA, in 1991 and 1993, respectively.

In 1989, he joined Drexel University, and until 1993, he conducted research in the development of thin- and thick-film high-temperature superconducting materials and devices and the development of ultralow-noise cryogenic characterization systems, where he was awarded a research fellowship by the Ben Franklin Superconductivity Center in 1989. From 1994 to 1996, he was a Principal Manager for R&D and the Director of a private sector research electrophysics laboratory while also teaching in the Departments of Electrical Engineering and Physics, Sharif University of Technology, Tehran. In 1996, he joined the Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey, teaching in the areas of solid state and electronics while also supervising his established Superconductivity Research Laboratory. In 1998 and 1999, he was invited to join the Forschungszentrum Jülich, Institut für Schicht und Ionentechnik, Jülich, Germany, where he pursued the development of low-noise high-Tc RF-SQUID-based magnetic sensors. In 2000, he established an international collaboration between Bilkent University, and Jülich Research Center, Jülich, in the field of superconductivity. From 2000 to 2004, he was the Director of the joint project for the development of a high-resolution high-Tc SQUID-based magnetic imaging system. Since 2000, he has also reestablished his activities in the Department of Electrical Engineering, Sharif University of Technology, where he is currently the Head of the Department of Electronics. In 2003, he set up the Superconductor Electronics Research Laboratory (SERL), Sharif University of Technology, which he has been directing since then. His research interest has mainly focused on the design, fabrication, and modeling of high-temperature superconducting devices and circuits such as bolometers, microwave filters and resonators, Josephson junctions, and SQUID-based systems, in the areas of which he holds several international patents.

Marko Banzet was born in Dinslaken, Germany, in 1973.

Since 1989, he has been with the Jülich Research Center (Forschungszentrum Jülich), Jülich, Germany, where he was a Physics Laboratory Assistant and has been a Technician since 1992, working in the field of superconductivity, thin-film deposition, ion beam etching, lithographic structuring, and clean room technology. He finished vocational training as an Information Technology Engineer in 2006.