Photonic band structure of Abrikosov lattices in superconductors

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ABSTRACT

We have performed a numerical solution for band structure of an Abrikosov vortex lattice in type-II superconductors forming a periodic array in two dimensions for applications of incorporating the photonic crystals concept into superconducting materials with possibilities for optical electronics. The implemented numerical method is based on the extensive numerical solution of the Ginzburg-Landau equation for calculating the parameters of the two-fluid model and obtaining the band structure from the permittivity, which depends on the above parameters and the frequency. This is while the characteristics of such crystals highly vary with an externally applied static normal magnetic field, leading to nonlinear behavior of the band structure, which also has nonlinear dependence on the temperature. The similar analysis for every arbitrary lattice structure is also possible to be developed by this approach as presented in this work. We also present some examples and discuss the results.

Keywords: Photonic crystal, Band Structure, Abrikosov Lattice.

1. INTRODUCTION

Recently, the concept of photonic band structure in type-II superconductors and its tunability and controllability with external parameters especially magnetic field are investigated[1,2] . In type-II superconductors when the applied external magnetic field is above the first critical field, a lattice of vortices is formed which depend on magnitude and direction of the applied magnetic field. Hence a permittivity contrast is obtained inside and outside of the vortices.

This periodic structure is in the form of a periodic distribution of permittivity normal to the superconductor slab, which represents a two-dimensional photonic crystal (2D-PC). So far the estimated calculation of band structure with simplified models for permittivity with step variation consideration has been reported[3].

Recently, an accurate approach of numerical computation of order parameters in Ginzburg-Landau equations with external magnetic field between the first and second critical magnetic fields is done[4]. By using this approach one can determine the continuous variation of permittivity along the superconductor. Generally the permittivity obtained is frequency-dependent, and thus the standard plane wave expansion (PWE) method would not be applicable to these calculations. More recently[5], a two-dimensional array of rods of superconductors is used to model a photonic crystal and its band structure is calculated numerically via plane wave expansion

Hence we have used the recently-reported revised plane wave expansion (RPWE)[6], which properly takes the effect of the dispersion and losses into account. By combining these two methods we have computed the precise band structure of the Abrikosov lattice. We have also compared these results with previous estimated calculations.

In a recent study by our group[4] we have reported the details of band-structure computations for the isotropic lattice formed by fluxons in low-T_c type-II superconductors. We showed that there is a dependency of gap to magnetic field. However, our study was limited to the case of low-T_c superconductors in which the Abrikosov lattice forms an isotropic 2D structure.

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Here, we report the extension of our latest research to the case of anisotropic Abirkosov lattice, which occurs usually in high-T_c superconductors.

The organization of this paper is as follows: in Sect. II we describe the theoretical model and method of numerical analysis used. In the next section the results and subsequent effects are discussed. In the last section we present the conclusions and summary.

2. THEORETICAL MODEL AND ANALYSIS

It can be shown that Maxwell's equations can be transformed into the following expression for harmonic magnetic modes which relates the permittivity (ε) to frequency (ω) and wave vector (κ):

$$\left[\nabla \times \frac{1}{\varepsilon(\mathbf{r})}\nabla\right] \times \mathbf{H}_{\omega}(\mathbf{r}) = \omega^2 \mathbf{H}(\mathbf{r}) . \tag{1}$$

In first step of this approach we choose TM mode analysis and simplify the above equation for z-axis electric field (E_z) and rewrite it in the form

$$\frac{\partial^2 E_z(x,y)}{\partial x^2} + \frac{\partial^2 E_z(x,y)}{\partial y^2} + \frac{\omega^2}{c^2}\varepsilon_{eff}(x,y;\omega)E_z(x,y) = 0 , \tag{2}$$

where ε_{eff} is the effective dielectric constant, which satisfies the above equation. This phenomenon is usual in electromagnetic theory, which helps us to find the desired parameters by previous approaches. In fact ε_{eff} consists of the principal information about the structure we are focusing on.

2.1 Implementing in superconducting materials

In type-II superconductors, when an external magnetic field is applied into the superconductor bulk, a periodic array of vortices is formed which allows the transmission of magnetic field through the body in a similar form likes cylinders. This is known as Meissner Effect, in the other hand in the way of magnetic field super-conducting electron pairs break and change to the normal-conducting electrons allowing the transmission of magnetic field, this phenomenon makes us using a model for this mix state; two-fluid model[8] explains the superconductor parameters by both normal-conducting and super-conducting electrons existing in any temperature bellow T_c

Hence, we can find the effective dielectric constant ε_{eff} with respect to electron densities by two-fluid model in all along the foresaid superconductors. Therefore ε_{eff} contains three parts; a non-dispersive dielectric constant ε_r , a superconducting lossless part and a normal-conducting one. It seems as follow:

$$\varepsilon_{eff} = \varepsilon_r \left\{1 - \frac{\omega_{ps}^2(x,y)}{\omega^2} - \frac{\omega_{pn}^2(x,y)}{\omega\left[\omega + i\gamma(x,y)\right]}\right\}, \tag{3}$$

where

$$\omega_{pn}(x,y) = \sqrt{\frac{n_n(x,y)e^2}{m\varepsilon_0\varepsilon}} \tag{4}$$

is the plasma frequency of normal-conducting electrons,

$$\omega_{ps}(x,y) = \sqrt{\frac{n_s(x,y)e^2}{m\varepsilon_0\varepsilon}} \tag{5}$$

is the plasma frequency of super-conducting electrons and $\gamma(x,y)$ is the damping term of normal-conducting electrons, which indicates the loss in superconductor materials. It is important that all of these parameters are considered locally inside the material, because the periodicity parameters are obtained in terms of them.

In general, ω_{ps} and ω_{pn} depend on superconducting n_s and normal electron densities n_n. It is obvious that the sum of these two densities is constant throughout, equaling to the total electron density:

$$n_n(x,y)+n_s(x,y)=n\,, \tag{6}$$

which means the square summation of normal and super-electoron plasma frequencies is also constant:

$$\omega_{pn}^2(x,y)+\omega_{ps}^2(x,y)=\omega_p^2\,. \tag{7}$$

Abrikosov vortex lattice is a 2-Dimensional array, which its parameters could be written as a 2-Dimensional Fourier series expansion; thus the effective dielectric constant is also periodic and could be written as

$$\varepsilon_r(\omega,\mathbf{r})=\sum_{m,n}\varepsilon_{mn}(\omega)\exp\left[j\left(G_{x,mn}\,x+G_{y,mn}y\right)\right], \tag{8}$$

where ε_{mn} is the coefficient of this Fourier expansion, which is obtained from effective dielectric constant as explained later, $G_{x,mn}$ and $G_{y,mn}$ are obtained from reciprocal lattice vectors:

$$G_{x,mn}\hat{x}+G_{y,mn}\hat{y}=m\frac{2\pi}{a}\hat{x}+n\frac{2\pi}{b}\hat{y}\,, \tag{9}$$

where a and b are lattice constants.

From Bloch Theorem we can write electric and magnetic field as a Bloch wave function:

$$\mathbf{E}(\mathbf{r})=\sum_{m,n}\mathbf{e}_{mn}\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}, \tag{10}$$

$$\mathbf{H}(\mathbf{r})=\sum_{m,n}\mathbf{h}_{mn}\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}, \tag{11}$$

where $\mathbf{e}_{mn}$ and $\mathbf{h}_{mn}$ are vectors which elements are coefficients for each direction. k_x and k_y are coefficients of wave vector in Brilliouin zone.

2.2 Ginzburg-Landau equation

Ginzburg-Landau equations are based on the 2-Dimensional average free energy per volume in superconductors, just like free energy in fluids, given by

$$\begin{aligned}\frac{F}{V}=f&=\left\langle-|\boldsymbol{\psi}|^2+\frac{|\boldsymbol{\psi}|^4}{2}+\left|\left(\frac{\nabla}{i\kappa}-\mathbf{A}\right)\boldsymbol{\psi}\right|^2+B^2\right\rangle\\&=\left\langle-\omega+\frac{\omega^2}{2}+\frac{(\nabla\omega)^2}{4\kappa^2\omega}+\omega\mathbf{Q}^2+(\nabla\times\mathbf{Q})^2\right\rangle\end{aligned}, \tag{12}$$

where $\boldsymbol{\Psi}$ is the GL order parameter[3], ω is the amplitude of $|\boldsymbol{\Psi}|^2$ (13), κ is the wave vector, $\mathbf{A}$ is the vector potential, $\mathbf{Q}$ is the supervelocity (14) and B is the coefficient of magnetic field in the normal direction of the surface. Meanwhile parameters have been normalized; B_c^2/μ_0 for f, $\sqrt{2}B_c$ for B and λ for lengths with $B_c=\Phi_0/(\sqrt{8}\pi\xi\lambda)$ the thermodynamic critical field, where ξ is the coherence length, λ is the magnetic penetration depth and Φ_0 is the quanta of magnetic flux in the superconductor bulk.

$$\boldsymbol{\Psi}(\mathrm{x,y})=\omega(\mathrm{x,y})^{1/2}\exp\left[i\phi(\mathrm{x,y})\right]. \tag{13}$$

$$\mathbf{Q}(x,y)=\mathbf{A}(x,y)-\frac{\nabla\varphi(x,y)}{\kappa}\,. \tag{14}$$

For calculating the effective dielectric constant we need to calculate the densities by the order parameter $\boldsymbol{\Psi}$ defined in Ginzburg-Landau equation (12). Here, one has three nonlinearly coupled equations in terms of the Fourier series coefficients of normalized $|\boldsymbol{\Psi}|^2$, magnetic field $\mathbf{B}$ and supervelocity $\mathbf{Q}$ for any position along the vortex photonic crystal. These three equations are

$$\omega(\mathbf{r}) = \sum_{m,n} a_{mn}\left(1-\cos(\mathbf{G}_{mn}.\mathbf{r})\right), \tag{15}$$

$$B(\mathbf{r}) = \overline{B} + \sum_{m,n} b_{mn}\cos(\mathbf{G}_{mn}.\mathbf{r}), \tag{16}$$

$$\mathbf{Q}(\mathbf{r}) = \mathbf{Q}_A(\mathbf{r}) + \sum_{m,n} b_{mn}\frac{\hat{z}\times\mathbf{G}_{mn}}{\left|\mathbf{G}_{mn}\right|^2}\sin(\mathbf{G}_{mn}.\mathbf{r}), \tag{17}$$

where $\mathbf{G}_{mn}$ is the reciprocal lattice vector, $\overline{B}$ is the DC external applied magnetic field and $\mathbf{Q}_A(\mathbf{r})$ is the supervelocity of the Abrikosov lattice when the magnetic field is in the second critical point (B_{c2}):

$$\mathbf{Q}_A(\mathbf{r}) = \frac{\nabla\omega_A\times\hat{z}}{2\kappa\omega_A}, \tag{18}$$

where $\omega_A(x,y)$ is the amplitude of $|\Psi|^2$ in the second critical magnetic field (B_{c2}):

$$a_{mn}^A = (-1)^{m+mn+n+1}\exp\left(-\frac{S\left|\mathbf{G}_{mn}\right|^2}{8\pi}\right), \tag{19}$$

where S is the unit cell area.

For solving the Ginzburg-Landau equation one has to minimize the free energy $f(a_{mn}, b_{mn})$ and find the stable answer in terms of Fourier coefficients. A solution is proposed by Brandt[3], which is to iterate the GL equation by using $\partial f/\partial\omega = 0$ and $\partial f/\partial Q = 0$ written in appropriate form like

$$(-\nabla^2 + 2\kappa^2)\omega = 2\kappa^2(2\omega - \omega^2 - \omega\left|\mathbf{Q}\right|^2 - g), \tag{20}$$

$$(-\nabla^2 + \overline{\omega})\mathbf{Q}_b = -\omega\,\mathbf{Q}_A - (\omega-\overline{\omega})\mathbf{Q}_b, \tag{21}$$

where

$$g = \frac{(\nabla\omega)^2}{4\kappa^2\omega}, \tag{22}$$

$$\overline{\omega} = \langle\omega\rangle = \sum_{m,n} a_{mn}, \tag{23}$$

is the average of the wave function over the unit cell area and

$$\mathbf{Q}_b = \mathbf{Q} - \mathbf{Q}_A. \tag{24}$$

For modeling the structure and simulating the super-electron wave function (ω) we have to limit the number of Fourier coefficients of GL equation parameters and consider the finite number of them and then applying the above approach; we will get three iteration equations as follow:

$$a_{mn} := \frac{4\kappa^2\left\langle(\omega^2 - 2\omega + \omega\mathbf{Q}^2 + g)\cos(\mathbf{G}_{mn}.\mathbf{r})\right\rangle}{\left|\mathbf{G}_{mn}\right|^2 + 2\kappa^2}, \tag{25}$$

$$a_{mn} := a_{mn}\frac{\left\langle\omega - \omega\mathbf{Q}^2 - g\right\rangle}{\left\langle\omega^2\right\rangle}, \tag{26}$$

$$b_{mn} := \frac{-2\left\langle [\omega B + \bar{\omega}(B-\bar{B}) + p]\cos(\mathbf{G}_{mn}.\mathbf{r})\right\rangle}{\left|\mathbf{G}_{mn}\right|^2 + \bar{\omega}}, \tag{27}$$

where

$$\begin{aligned} p &= (\nabla\omega \times \mathbf{Q})\,\hat{z} \\ &= Q_x \frac{\partial\omega}{\partial y} - Q_y \frac{\partial\omega}{\partial x}. \end{aligned} \tag{28}$$

For stable and fast convergence one can initialize the values of $a_{mn} = a^A_{mn}$ and b_{mn}= 0. After iterating (25) and (26) for few times to relax $\omega(x,y)$, we will use all three equations (25-27) for iterating. For better approximation one could use the iteration more times getting the desired error.

One can test the algorithm or find the error by using the orthogonality of the functions:

$$a_{mn} = -2\left\langle \omega(\mathbf{r})\cos(\mathbf{G}_{mn}.\mathbf{r})\right\rangle, \tag{29}$$

$$b_{mn} = \ 2\left\langle B(\mathbf{r})\ \cos(\mathbf{G}_{mn}.\mathbf{r})\right\rangle, \tag{30}$$

then comparing the coefficients with previous ones. Meanwhile the difference between values of consequent iterations could show the error.

2.3 Building permittivity

From previous section one has Fourier series expansion of wave function (super-conducting electrons); one must insert it into the effective dielectric constant introduced in section 2.1, using (7) we rewrite (3) as follow:

$$\varepsilon_{eff} = \varepsilon_r \left\{ 1 - \frac{\omega^2_{ps}(x,y)}{\omega^2} - \frac{\omega^2_p - \omega^2_{ps}(x,y)}{\omega\left[\omega + i\gamma(x,y)\right]} \right\}. \tag{31}$$

After simplifying one gets:

$$\varepsilon_{eff} = \varepsilon_r \left\{ 1 - \frac{\omega^2_p}{\omega\left[\omega + i\gamma(x,y)\right]} - \omega^2_{ps}(x,y)\left(\frac{i\gamma(x,y)}{\omega^2\left[\omega + i\gamma(x,y)\right]} \right) \right\}. \tag{32}$$

In such large frequencies working in superconductor materials, variation of damping term γ over the entire unit cell is almost negligible, which means it could be considered constant, therefore $\gamma(x,y) \approx \gamma$.

The permittivity matrix could be written as

$$\left[\varepsilon_{eff}\right] = \varepsilon_r \left(1 - \frac{\omega^2_p}{\omega\left[\omega + i\gamma\right]} \right)\left[\mathbf{I}\right] - \varepsilon_r \left(\frac{i\gamma}{\omega^2\left[\omega + i\gamma\right]} \right)\left[\omega^2_{ps}(x,y)\right]. \tag{33}$$

Considering (5) it will turn to

$$\left[\varepsilon_{eff}\right] = \varepsilon_r \left(1 - \frac{\omega^2_p}{\omega\left[\omega + i\gamma\right]} \right)\left[\mathbf{I}\right] - \varepsilon_r \left(\frac{i\gamma}{\omega^2\left[\omega + i\gamma\right]} \right)\left(\frac{e^2}{m\varepsilon_0\varepsilon} \right)\left[n_s(x,y)\right], \tag{34}$$

where $n_s(x,y)$ is also named as wave function $\omega(x,y)$ introduced in section 2.2, hence one has the Fourier series expansion of permittivity obtained from wave function of super-electron density solved by iterating Ginzburg-Landau equation.

Normalization factors must be considered carefully; in section 2.2, $\omega(x,y)$ was normalized, hence ω^2_p has to be inserted as 1 and coefficients of $n_s(x,y)$ must be eliminated, therefore (34) will be written as

$$\left[\varepsilon_{eff}\right]=\varepsilon_r\left(1-\frac{1}{\omega\left[\omega+i\gamma\right]}\right)\left[\mathbf{I}\right]-\varepsilon_r\left(\frac{i\gamma}{\omega^2\left[\omega+i\gamma\right]}\right)\left[\omega(x,y)\right]. \tag{35}$$

2.4 Revised Plane Wave Method (RPWM)

For considering the dispersion effect of permittivity we use the revised plane wave expansion method (RPWE) while the Fourier series coefficient of permittivity is obtained by previous section. In RPWE the normalized frequency f and wave-vector in x-direction κ_x are scanned and κ_y is obtained as the eigenvalues of a matrix, which is a result of combining Maxwell's equations. The Photonic Band Gap (PBG) depends on the distances between vortices, as well as the intensity of applied magnetic field. The radius of vortices is almost constant and is proportional to magnetic penetration depth λ, which is related to the material, used as a superconductor; hence this parameter is not an easy variable like ordinary photonic crystals. The resulting periodic perturbations of the permittivity give rise to a bandstructure, which PBGs and cut-off frequencies vary with the applied magnetic field.

In 2-D photonic crystals such as discussed in this paper we could consider both TE and TM polarizations, we write the Maxwell's equations for each of them and try to obtain an eigenvalue equation, which their eigenvalue is κ_x or κ_y as explained above.

We start from TE mode; the well-known Maxwell's equations are written as

$$\begin{aligned}\frac{\partial H_z}{\partial y}&=j\omega\varepsilon_0\varepsilon_r(\omega)E_x\\ -\frac{\partial H_z}{\partial x}&=j\omega\varepsilon_0\varepsilon_r(\omega)E_y\\ \frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}&=j\omega\mu_0 H_z\end{aligned}. \tag{36}$$

Electric and Magnetic fields could be written as

$$\mathbf{E}(\mathbf{r})=\sum_{m,n}\left(\hat{x}e_{x,mn}+\hat{y}e_{y,mn}\right)\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}, \tag{37}$$

$$\mathbf{H}(\mathbf{r})=\hat{z}\frac{1}{Z_0}\sum_{m,n}h_{z,mn}\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}. \tag{38}$$

We must change (36) to the matrix form to obtain an eigenvalue equation, for that purpose we show how to change the first equation of (36), one has to substitute (8), (37) and (38) in first equation of (36):

$$\begin{aligned}&\frac{\partial}{\partial y}\left\{\frac{1}{Z_0}\sum_{m,n}h_{z,mn}\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}\right\}=\\ &\qquad j\omega\varepsilon_0\left\{\sum_{m,n}\varepsilon_{mn}(\omega)\exp\left[j\left(G_{x,mn}x+G_{y,mn}y\right)\right]\right\}\times\left\{\sum_{m,n}e_{x,mn}\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}\right\}\end{aligned}. \tag{39}$$

After derivation of the left hand side, it will be

$$\begin{aligned}&j\frac{1}{Z_0}\sum_{m,n}h_{z,mn}\left(G_{y,mn}+k_y\right)\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}=\\ &\qquad j\omega\varepsilon_0\left\{\sum_{m,n}\varepsilon_{mn}(\omega)\exp\left[j\left(G_{x,mn}x+G_{y,mn}y\right)\right]\right\}\times\left\{\sum_{m,n}e_{x,mn}\exp\left\{j\left[\left(G_{x,mn}+k_x\right)x+\left(G_{y,mn}+k_y\right)y\right]\right\}\right\}\end{aligned}. \tag{40}$$

We use the Balance Method to relate each multiplicand of exponential terms. After multiplying two series of the right hand side, we must rearrange them in the way; similar exponential terms appear in each side. Then each exponential function multiplicand occupies a distinct position in matrices relating the magnetic field to the electric field. The right hand side of the above equation could be rearranged as bellow:

$$j\omega\varepsilon_0\left\{\sum_{m,n,p,q}\varepsilon_{m,n}(\omega)e_{x,p,q}\exp\left[j\left(G_{x,m,n}x+G_{y,m,n}y\right)\right]\exp\left\{j\left[\left(G_{x,p,q}+k_x\right)x+\left(G_{y,p,q}+k_y\right)y\right]\right\}\right\}$$
$$=j\omega\varepsilon_0\left\{\sum_{m,n,p,q}\varepsilon_{m,n}(\omega)e_{x,p,q}\exp\left\{j\left[\left(G_{x,m+p,n+q}+k_x\right)x+\left(G_{y,m+p,n+q}+k_y\right)y\right]\right\}\right\} \quad (41)$$

Defining $r=m+p$ and $s=n+q$, above expression turns to

$$j\omega\varepsilon_0\left\{\sum_{m,n,r,s}\varepsilon_{m,n}(\omega)e_{x,r-m,s-n}\exp\left\{j\left[\left(G_{x,r,s}+k_x\right)x+\left(G_{y,r,s}+k_y\right)y\right]\right\}\right\}. \quad (42)$$

Here we have similar terms in each side of (40) as following:

$$\left\{j\frac{1}{Z_0}\sum_{r,s}h_{z,rs}\left(G_{y,rs}+k_y\right)\exp\left\{j\left[\left(G_{x,rs}+k_x\right)x+\left(G_{y,rs}+k_y\right)y\right]\right\}\right\}=$$
$$j\omega\varepsilon_0\left\{\sum_{m,n,r,s}\varepsilon_{m,n}(\omega)e_{x,r-m,s-n}\exp\left\{j\left[\left(G_{x,rs}+k_x\right)x+\left(G_{y,rs}+k_y\right)y\right]\right\}\right\}. \quad (43)$$

We could now equalize the multiplicand of each corresponding term using Balance Method:

$$h_{z,rs}\left(G_{y,rs}+k_y\right)=\omega\varepsilon_0\sum_{mn}\varepsilon_{mn}(\omega)e_{x,r-m,s-n}\ . \quad (44)$$

Therefore the matrix form of equations (36) in a similar way, will be as

$$\begin{aligned}\left(k_y+\left[G_y\right]\right)\left[h_z\right]&=\ k_0\left[\kappa\right]\left[e_x\right]\\ \left(k_x+\ \left[G_x\right]\right)\left[h_z\right]&=-k_0\left[\kappa\right]\left[e_y\right]\\ \left(k_y+\left[G_y\right]\right)\left[e_x\right]-\left(k_x+\left[G_x\right]\right)\left[e_y\right]&=k_0\left[h_z\right]\end{aligned}, \quad (45)$$

where $k_0=\omega/c$, $[e_x]$, $[e_y]$ and $[h_z]$ are three column vectors in the form bellow, these are Fourier coefficients and describe the electric and magnetic fields respectively:

$$\left[e_x\right]=\begin{bmatrix}e_{x11}\\e_{x21}\\\vdots\\e_{x12}\\e_{x22}\\\vdots\end{bmatrix},\qquad \left[e_y\right]=\begin{bmatrix}e_{y11}\\e_{y21}\\\vdots\\e_{y12}\\e_{y22}\\\vdots\end{bmatrix},\qquad \left[h_z\right]=\begin{bmatrix}h_{z11}\\h_{z21}\\\vdots\\h_{z12}\\h_{z22}\\\vdots\end{bmatrix}. \quad (46)$$

Therefore the reciprocal lattice vectors $[G_x]$ and $[G_y]$ must be in the following format:

$$G_x=\begin{bmatrix}G_{x11}&0&\cdots&0&0&\cdots\\0&G_{x21}&&0&0&\\\vdots&&\ddots&\vdots&\vdots&\\0&0&\cdots&G_{x12}&&\\0&0&\cdots&&G_{x22}&\\\vdots&&&&&\ddots\end{bmatrix}, \quad (47)$$

$$G_y = \begin{bmatrix} G_{y11} & 0 & \cdots & 0 & 0 & \cdots \\ 0 & G_{y21} & & 0 & 0 & \\ \vdots & & \ddots & \vdots & \vdots & \\ 0 & 0 & \cdots & G_{y12} & & \\ 0 & 0 & \cdots & & G_{y22} & \\ \vdots & & & & & \ddots \end{bmatrix}. \tag{48}$$

But it is however more difficult to obtain $[\kappa]$ because of its irregular form in two dimensions. It could be done by considering (44) & (46); result in a matrix, which is occupied by permittivity elements in order of difference between indexes.

2.5 Eigenvalue Problem

Here, we use (45) to find the eigenvalue equation, which its eigenvalue is k_x or k_y, this helps us to rearrange the equations of (45) in the way, k_y is a multiplicand of electric or magnetic field, after few math works we reach to

$$\begin{bmatrix} -[G_y] & k_0 - (k_x + [G_x])[\kappa]^{-1}(k_x + [G_x])/k_0 \\ k_0[\kappa] & -[G_y] \end{bmatrix} \begin{bmatrix} [e_x] \\ [h_z] \end{bmatrix} = k_y \begin{bmatrix} [e_x] \\ [h_z] \end{bmatrix}. \tag{49}$$

If one is interested to use k_x as an eigenvalue, it is possible to rearrange it to get the result.

The approach for TM mode is the same; here is the eigenvalue equation for TM mode:

$$\begin{bmatrix} -[G_y] & (k_x + [G_x])^2/k_0 - k_0[\kappa] \\ -k_0 & -[G_y] \end{bmatrix} \begin{bmatrix} [h_x] \\ [e_z] \end{bmatrix} = k_y \begin{bmatrix} [h_x] \\ [e_z] \end{bmatrix}. \tag{50}$$

It is obvious that considering dispersion leads to double size of eigenvalue matrix and the algorithm needs more time.

3. RESULTS AND DISCUSSION

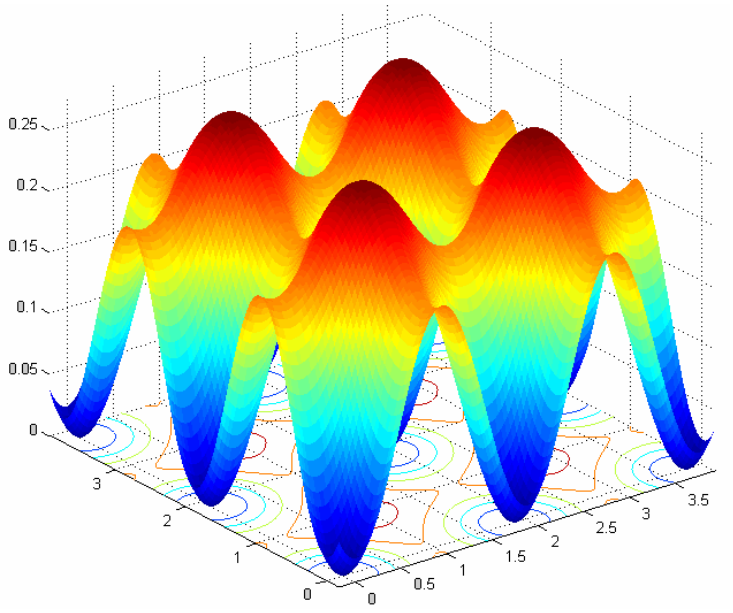

Fig. 1. Distribution of the normalized superelectron density versus two-dimensional position for κ=1.5 and **B**=0.7.

Fig. 1 shows a typical result for Ginzburg-Landau solution. It shows $|\Psi|^2 = n_s/n$ versus position in the superconductor. By solving the recursive equations, the Fourier expansion coefficient of superconducting electron densities is obtained and then both plasma frequencies are computed as shown in section 2.1.

Having the permittivity known with respect to the position in superconductor, the corresponding Fourier coefficients can be obtained. In our calculation method, the coded algorithm can be equally applied to both Standard Square and triangular lattice geometries. Then the series coefficients are applied to bandstructure calculations.

Damping term, applied magnetic field and Ginzburg-Landau parameter are the three major parameters that affect the band structure. For high damping term, above the scanned frequencies, variation of permittivity decreases and thus we have lower cut-off frequencies.

Here we have done our analysis with damping term that is much more than strictly analyzing frequency. Thus the results could be compared with previous estimated ones.

As it is apparent from Fig. 2, the cut-off frequency strongly depends on the magnetic field and with increasing the magnetic field the cut-off frequency decreases. It is also concluded that in precise calculation, falling rate for cut-off frequency is lower than that of approximate calculations for a given value of γ.

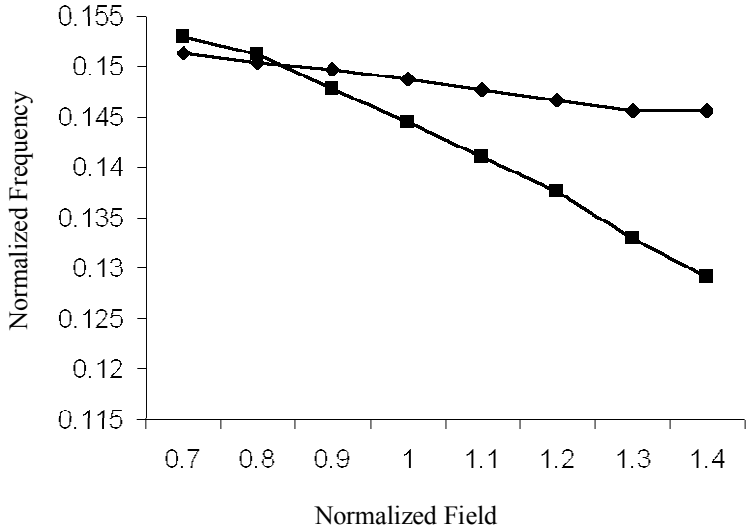

Fig. 2. Estimated (Squares) and precise (Diamonds) results for variations of normalized cut-off frequency versus magnetic field.

For anisotropic structures a new improvement is presented[7], which introduced an anisotropic coefficient in the previous recursive equations, we included this coefficient and did our approach for these structures again. In Fig. 3 the wave function for this anisotropic crystal has been presented.

The Contours for isotropic an anisotropic structure is presented in Fig. 4 and Fig.5.

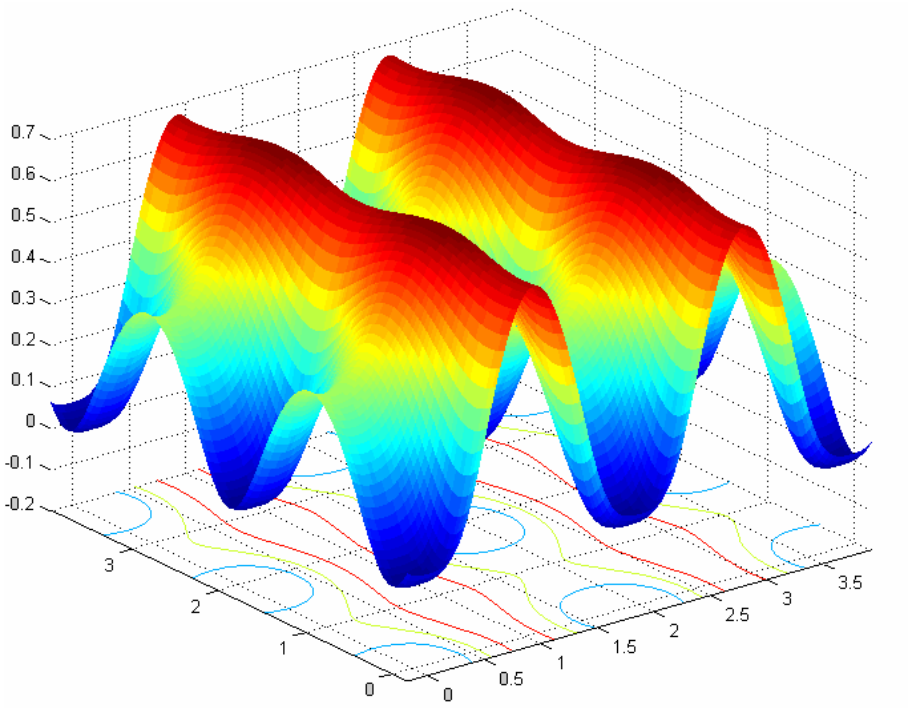

Fig. 3. Distribution of the normalized superelectron density for κ=1.5 and **B**=0.7 in anisotropic structure.

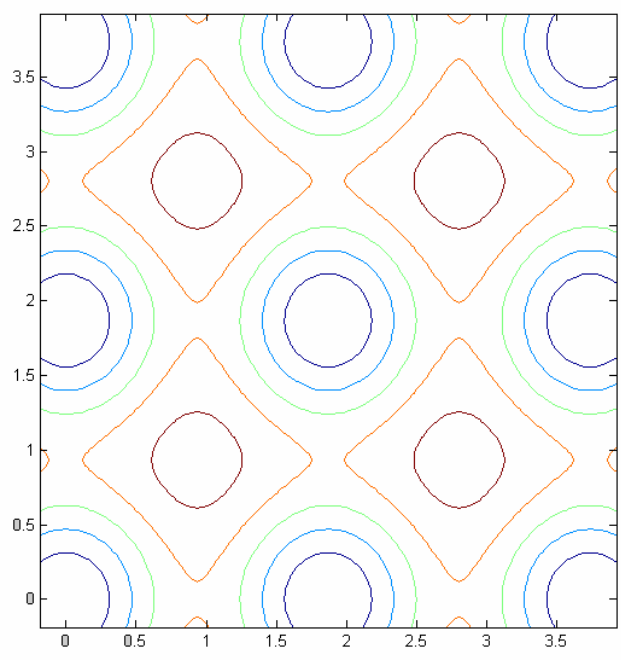

Fig. 4. Contours for distribution of the normalized superelectron density for κ=1.5 and **B**=0.7 in isotropic structure.

As it is obvious in non-isotropic case according to existence of copper oxide layers between superconducting regions Abrikosov lattice exhibits asymmetric behavior across the crystal axes, therefore for high T_c superconductors that exhibit this type of anisotropy, we have a new feature that photonic band structure shows two different behaviors across crystal axes.

For solving equations (12-28) in the non-isotropic case, a matrix (**Γ**) is introduced[7], which one could insert it in the above equations and redo the same recursive procedure.

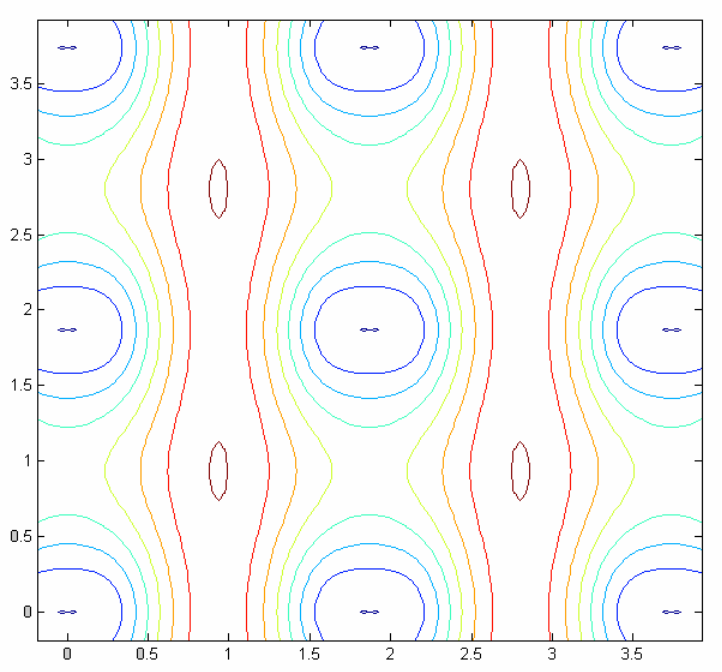

Fig. 5. Contours for distribution of the normalized superelectron density for κ=1.5 and **B**=0.7 in anisotropic structure with anisotropic coefficient, $\alpha = 0.25$.

Similar to the case of isotropy, we apply revised plane wave method to result of this process and extract the photonic band structure. The **Γ** matrix reads

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}, \tag{51}$$

where α is anisotropy factor[7] and defined as m_x / m_y . The band-structure and its dependency to magnetic field and other parameters will be presented later.

4. CONCLUSION

We solved the coupled Ginzburg-Landau and Photonic Band Gap equations, with full accuracy based on two-fluid model, with taking the effect of anisotropy into account. A general code was developed capable of solving for arbitrary geometry and set conditions. We noticed that the cut-off frequency decreases significantly with the applied magnetic field. This effect provides a window to novel potential applications such as precision magneto-optical tunable filters and modulators. The dependence of other parameters of the band structure on the applied field and the vortex lattice is under further investigation.

REFERENCES

1. C. H. R. Ooi, T. C. A. Yeung, C. H. Kam and T. K. Lim, "Photonic band gap in a superconductor-dielectric superlattice," *Phys. Rev. B* 61, 5920 (2000).
2. H. Takeda and K. Yoshino, "Properties of Abrikosov lattices as a photonic crystals," *Phys. Rev. B* 70, 085109 (2004).
3. E.H. Brandt, "Precision Ginzburg-Landau solution of ideal vortex lattices for any induction and symmetry," Phys. *Rev. Let.* 78, 2208 (1997).
4. A. Kokabi, H. Zandi, S. Khorasani, and M. Fardmanesh, "Precision photonic band structure calculation of Abrikosov periodic lattice in type-II superconductors," *8th International Conference on Materials and Mechanisms of Superconductivity – High Temperature Superconductors, Dresden* (2006).
5. O. L. Berman, Y. E. Lozovik, S. L. Eiderman and R. D. Coalson, "Superconducting photonic crystals: Numerical calculations of the band structure," *Phys. Rev. B* 74, 92505 (2006).
6. Sh. Shi, C. Chen, D. W. Prather, "Revised plane wave method for dispersive material and its application to band structure calculations of photonic crystal slabs," *Appl. Phys. Let.* 86, 043104 (2005).
7. A. Achalere and B. Dey, "Anisotropic Ginzburg-Landau theory for arbitrary induction and vortex lattice symmetry: Effects of a-b plane mass anisotropy on the properties of high-temperature superconductors," *Phys. Rev. B* 71, 224504 (2005).
8. H. Takeda and K. Yoshino, "Tunable photonic band schemes in two-dimensional photonic crystals composed of copper oxide high-temperature superconductors" *Phys. Rev. B* 67, 245109 (2003).