Mechanical Force Optimization on Superconductor Element of Shield-type Superconducting FCLs

Arsalan Hekmati         Mehdi Vakilian         Mehdi Fardmanesh
Electrical Engineering Department
Sharif University of Technology
Tehran, Iran

Abstract— Superconducting fault current limiter (SFCL) is classified in the category of most efficient fault current limiters. Inductive shield-type SFCLs use bulk superconductor rings in their structure. At the fault occurrence, the induced current in the superconductor ring causes high mechanical forces on the ring. In the preliminary fabricated prototypes of this work, this mechanical force caused several damages to the superconductor element. Thus in this paper an analytical method is introduced to yield the main design parameters of the shield-type SFCL to ensure the sufficient mechanical withstand capability of the superconductor ring employed in this design. A prototype SFCL of this type is fabricated in this work (using the superconductor ring parameters, determined through this analytical method) to limit the fault current to about 2 A. The fabricated device is tested and no damage occurred on the superconducting ring.

Keywords- fault current limiter; superconductor; shield type; mechanical force

I. INTRODUCTION

The superconducting fault current limiter (SFCL) can be used to limit the short-circuit current level in electrical transmission and distribution networks and is one of the most promising devices for application in the transmission and distribution networks of electrical energy systems due to its low nominal losses, reliable operation, very short reaction times to fault currents, and an automatic response feature without the requirement of external trigger mechanism [1].

An inductive current limiter works like a transformer that has a shorted superconducting secondary winding. The impedance of this current limiter under normal operating conditions is nearly zero, since the zero impedance of the secondary superconducting winding is reflected on the primary circuit. In the event of a fault, the secondary superconducting winding fails to be superconducting. Subsequently, the secondary resistance will increase and limit the fault current [2].

The secondary winding is usually an HTS bulk cylinder whose function under normal condition is to shield the flux generated by the primary winding from entering the iron core of the current limiter. The primary winding is usually made from copper connected directly to an electric circuit. In the normal operation of the SFCL, according to Meissner effect, no flux enters the core and the SFCL shows a low inductance. In the fault condition the ampere-turns balance of the transformer is destroyed and flux from the primary winding enters the iron core. The inductance and impedance of the primary winding rapidly increase limiting the fault current of the circuit [3].

The cut-away view of an inductive shield-type SFCL is shown in Fig. 1.

A laboratory scale SFCL, with this structure, was fabricated in this work. During tests of this device (while it is employed in a sample circuit) under application of different fault conditions, (in some cases) the superconductor ring experienced some cracks which diminished its current limitation capability. Thus a design process was investigated to reduce the mechanical forces on the superconducting ring during its operation. The ring designed and fabricated under the proposed process (i.e. specific dimensions), employed in fabrication of another SFCL prototype which demonstrated more satisfactory mechanical behavior.

I. OPERATION OF SFCL

As the SFCL structure (shown in Fig. 1) has an axial symmetry, a two-dimensional model of this inductive shield-type SFCL is shown in Fig. 2.

Fig. 1. Cut-away view of the inductive SFCL [4]

Figure 1

Figure 2

[4]
The main design variables of this type of SFCL (shown in Fig. 2) are given in the following:

- $L_1$: the length of the copper winding
- $L_2$: the length of the bulk superconducting tube
- $N$: the number of turns of the copper winding
- $r_{2i}$: the interior radius of the superconducting bulk
- $r_{2o}$: the exterior radius of the superconducting bulk
- $r_{1i}$: the interior radius of the copper winding
- $r_{1o}$: the exterior radius of the copper winding
- $rc$: radius of the core

Two attributes of the SFCL are known as:

- $I_{\text{start}}$: the activation current in which the inductance starts to increase
- $I_{\text{limit}}$: the specific magnitude of current at which the current is limited

### A. SFCL Limitation characteristics

In [5] the maximum amplitude of the limited current is obtained to be as (1).

$$I_{\text{limit}} = I_{\text{start}} + \frac{V_m L_1}{\pi \mu_0 N^2 (\mu - 1) r_{2i}^2}$$

(1)

Where $I_{\text{start}}$ is calculated from (2).

$$I_{\text{start}} = \frac{J_c L_1 (r_{2o} - r_{2i})}{N}$$

(2)

The superconducting FCL has the advantage of limiting the fault current reliably to a predetermined value of $I_{\text{limit}}$. Using an SFCL in the circuit, it can be assured that the system current would not exceed $I_{\text{limit}}$. The magnitude of $I_{\text{limit}}$ decreases sharply with an increase in the internal radius of the bulk superconductor and with an increase in the turn’s number of the copper winding. It increases when the bulk thickness and the length of the copper winding increase. A criterion for the Relative Overshoot of the Current over $I_{\text{start}}$, ROC, may be defined by relation (3).

$$ROC = \frac{I_{\text{limit}} - I_{\text{start}}}{I_{\text{start}}} = \frac{V_m}{\pi \mu_0 N J_c (\mu - 1) r_{2o}^2 (r_{2o} - r_{2i})}$$

(3)

ROC is a criterion of how much the current has exceeded the activation current, $I_{\text{start}}$. For a given voltage level, the current overshoot would decrease with increasing the turns number of the copper winding, the interior radius and the thickness of the superconductor bulk.

The shield-type SFCL has two operation regimes. One regime corresponds to the normal operating phase and the other regime corresponds to the fault current limiting phase. The flux density distribution in the superconducting bulk in these two phases is as Fig. 3, and the current density in the superconductor bulk is as Fig. 4, [5].

## II. FORCES ON THE BULK SUPERCONDUCTOR

The radial force imposed on the bulk superconductor ring is calculated for its two operation regimes. The radial forces may be calculated using (4) [6].

$$dF = I (d\ell \times \hat{B})$$

(4)

The superconducting ring may be assumed as cylindrical shells of thickness equal to ($dr$). The current in each of these shells would be $J_c L_2 dr$. In a cylindrical coordination

![Fig. 2. The schematic arrangement of an inductive SFCL](image)

![Fig. 3. Flux density distribution a) in the Normal operating phase b) in the current limiting phase of the shield-type SFCL operation](image)

![Fig. 4. Current density distribution a) in the Normal operating phase b) in the current limiting phase of the shield-type SFCL](image)
assuming the central axis of the ring and the magnetic field along the z axis, the current would be in φ direction.

The magnetic field produced by the copper winding may be assumed to be constant in the location of superconducting ring (since its height is much shorter than the copper winding height). Thus this magnetic flux density is calculated from (5).

\[
B = \frac{\mu_0 NI}{L_1}
\]  

(5)

The magnetic flux density inside the ring has a distribution dictated by the Bean Model (shown in Fig. 3), which follows the relation of (6), [7].

\[
\frac{dB}{dr} = \mu_0 J_c \mu_0 J_c
\]  

(6)

Thus according to (5) and (6) and the flux density in Fig. 3, the radial force which is imposed on an angular portion of the shell (dφ) during its normal operating phase, as shown in Fig. 5, can be obtained from relation (7).

\[
dF = \int_J \left(J_c L_2 d\phi \alpha \left[\mu_0 J_c (r - r_{z_0}) + \frac{\mu_0 NI}{L_1}\right]\right) d\phi
\]  

(7)

Which yields (8).

\[
dF = d\phi J_c L_2 \left[\frac{1}{2} \left(\frac{\mu_0 NI}{L_1} - \mu_0 J_c r_{z_0}\right) (r_{z_0}^2 - (r')^2)
\right. \\
\left. + \frac{1}{3} \mu_0 J_c (r_{z_0}^3 - (r')^3)\right] dr
\]  

(8)

Where; \(r' = r_{z_0} - \frac{NI}{L_1 J_c}\) is the radius at which the flux density reaches zero inside the superconductor bulk.

In the current limiting phase, since the flux passes through the superconductor ring and penetrates into the core, it is sufficient to replace \(r'\) with \(r_{z_0}\). Thus the radial force can be determined by relation (9).

\[
dF = d\phi J_c L_2 \left[\frac{1}{2} \left(\frac{\mu_0 NI}{L_1} - \mu_0 J_c r_{z_0}\right) (r_{z_0}^2 - r_{z_2}^2)
\right. \\
\left. + \frac{1}{3} \mu_0 J_c (r_{z_0}^3 - r_{z_2}^3)\right] dr
\]  

(9)

According to the equations (8) and (9), the maximum mechanical force occurs in the current limiting phase of SFCL operation under fault conditions. Thus it is sufficient to minimize only the applied force that can be calculated by equation (9). However, the important factor which plays role in magnitude of stresses imposed on the bulk superconductor is not force, but it is the force per unit area (pressure) which can be calculated for the interior and the exterior surface of the angular portion (dφ) of the ring (shown in Fig. 5). For the interior surface of the ring, the pressure may be determined from (9) as (10).

\[
P_{in} = \frac{d\phi J_c L_2 \left[\frac{1}{2} \left(\frac{\mu_0 NI}{L_1} - \mu_0 J_c r_{z_0}\right) (r_{z_0}^2 - r_{z_2}^2)
\right. \\
\left. + \frac{1}{3} \mu_0 J_c (r_{z_0}^3 - r_{z_2}^3)\right] }{L_2 (r_{z_2} d\phi)}
\]  

(10)

This yields (11).

\[
P_{in} = \frac{J_c \left[\frac{1}{2} \left(\frac{\mu_0 NI}{L_1} - \mu_0 J_c r_{z_0}\right) (r_{z_0}^2 - r_{z_2}^2) + \frac{1}{3} \mu_0 J_c (r_{z_0}^3 - r_{z_2}^3)\right] }{r_{z_2}}
\]  

(11)

By application of analogy, the tension on the outer surface of the ring, can be determined through equation (12);

\[
P_{out} = \frac{J_c \left[\frac{1}{2} \left(\frac{\mu_0 NI}{L_1} - \mu_0 J_c r_{z_0}\right) (r_{z_0}^2 - r_{z_2}^2) + \frac{1}{3} \mu_0 J_c (r_{z_0}^3 - r_{z_2}^3)\right] }{r_{z_0}}
\]  

(12)

Since the limited current exceeds \(I_{start}\) by an amount equal to \(ROC \times I_{start}\), therefore \(I_{limit}\) is obtained from (13).

\[
I_{limit} = I_{start} (1 + ROC)
\]  

(13)

This is the maximum current which the SFCL experiences. Since the pressure on the inner surface of the ring (obtained by relation 11) is greater than the pressure on the outer surface (obtained by relation 12), the required criterion should be imposed on the inner surface of the ring. Therefore, in relation (12), \(I\) is replaced by its maximum value in (13). In relation (14) the maximum pressure on the surfaces of the superconductor ring is obtained.

\[
P_{max} = \frac{J_c \left[\frac{1}{2} \left(\frac{\mu_0 NI}{L_1} - \mu_0 J_c r_{z_0}\right) (1 + ROC - \mu_0 J_c r_{z_0}) (r_{z_0}^2 - r_{z_2}^2)
\right. \\
\left. + \frac{1}{3} \mu_0 J_c (r_{z_0}^3 - r_{z_2}^3)\right] }{r_{z_2}}
\]  

(14)

Fig. 5. The interior and the exterior surface of the angular portion dφ of the superconductor ring
III. RESULTS AND ANALYSIS

From (14) it’s apparent that the height of the superconductor ring would not impact the pressure on the bulk. Only the impact of $r_{20}$ and $r_{2i}$ should be determined. Variation of the pressure on the superconducting ring versus the internal radius of the superconducting ring and its thickness are shown in Fig. 6, and Fig. 7, respectively. In Fig. 6, the internal radius of the superconducting ring has been assumed constant and the effect of the ring thickness has been showed. In Fig. 7, the thickness of the ring has been set constant and the pressure versus the internal radius of the ring is plotted. The variation in the volume of the superconductor ring is included in both figures.

It may be concluded that decreasing the thickness of the superconductor ring results in the decrement change in the objective function $P$, as Fig. 6, therefore lower thickness of the superconductor ring is more desirable.

The limiting factors in the decrement change of the superconductor ring thickness are the fabrication limitations. For example a casting method has been chosen in this work for the bulk superconductor fabrication and the limiting factor for the ring thickness is the dimensions of the cast used in the ring production.

Increasing the internal radius of the superconductor ring, results in the decrease of the pressure on the ring too, as Fig. 7. Here the limiting factors are the width of the furnace nozzle and the increased volume of the superconductor ring as the volume of the ring increases with increasing both the internal radius of the ring and its thickness, as shown in Fig. 6, and Fig 7.

A. Dimensions of the superconductor ring

For the process which has been chosen in this work for the bulk superconductor fabrication, there is a minimum for the bulk thickness dictated by the minimum dimensions of the cast used for the shaping of the ring. These minimum dimensions are required to provide the necessary pressure force withstand capability. These dimensions dictate at least 2 mm thickness for the superconductor ring.

As stated, the higher interior radius results in lower pressure on the superconductor ring surface. Nevertheless, there are two limitations for increasing this radius. One is the increasing volume of the bulk as shown in Fig. 7. The other limitation encountered in this work is the width of the furnace nozzle. This nozzle width limits the maximum available exterior radius of the superconductor bulk. In this work the nozzle width is 35mm. Thus the external diameter of the bulk may not exceed this value. As the height of the bulk acts as a limitation in the entrance of the ring to the furnace, taking into account the height of the bulk lowers the maximum possible external radius of the superconductor ring, therefore a lower value for example 34mm for the external diameter is chosen, thus the internal radius of the bulk should not exceed the value of $r_{2i} = (34 - 2 \times 2)/2 = 15mm$.

The critical current of the bulk superconductor is measured to be approximately $J_c = 9.5 \times 10^6$ A/m$^2$ in this work. Thus the activation current in which the limitation process is desired to start may be calculated from (2), which yields an activation current of 2.2A.

B. Fabricated prototype

A prototype with the obtained dimensions for the superconductor ring and the other design parameters as in table I, has been fabricated and tested to verify the required attributes. It’s shown in Fig. 8.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Design parameters of the SFCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal radius of the flat superconductor ring</td>
<td>15 mm</td>
</tr>
<tr>
<td>External radius of the flat superconductor ring</td>
<td>17 mm</td>
</tr>
<tr>
<td>Height of the flat superconductor ring</td>
<td>4 mm</td>
</tr>
<tr>
<td>Turns number of the copper winding</td>
<td>35</td>
</tr>
<tr>
<td>Diameter of the copper wire</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>Internal radius of the copper winding</td>
<td>18 mm</td>
</tr>
<tr>
<td>External radius of the copper winding</td>
<td>19 mm</td>
</tr>
<tr>
<td>Height of the copper winding</td>
<td>40 mm</td>
</tr>
<tr>
<td>Radius of the core</td>
<td>13 mm</td>
</tr>
<tr>
<td>Relative permeability of the core</td>
<td>500</td>
</tr>
</tbody>
</table>
The whole device is immersed in the liquid nitrogen with temperature 77 °k for cooling.

The V-I characteristic of the device was measured via imposing different voltages to the SFCL terminals and recording the through current. The result is shown in Fig. 9. The activation current in which the inductance of the SFCL starts to increase is approximately 2 A, as shown in this figure. No damages occurred on the superconductor ring during these tests.

**IV. SUMMARY AND CONCLUSION**

Based on the introduced process, in the shield-type superconducting fault current limiters, the mechanical forces on the bulk ring may be calculated and the dimensions may be chosen for the superconductor ring in order to insure that the bulk would experience acceptable and tolerable mechanical forces during its current limitation process. Thus the probability of the bulk mechanical breakdown during its operation will be minimized. The phenomenon is reported in several documents.

An optimization process may be conducted to achieve minimization of several target functions. One of them is the mechanical forces. Especially as described in this paper, reconciliation may be considered between the mechanical forces and the volume of the superconductor bulk, resulting in optimization of superconductor material consumption.

**V. REFERENCES**


